RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

FIRST YEAR

B.A./B.SC. FIRST SEMESTER (July – December), 2012

Mid-Semester Examination, September 2012

Date : 10/09/2012

Time : 11 am – 1 pm

PHYSICS (Honours)

Paper : I

Full Marks : 50

Use two Answer Scripts. One for each section.

SECTION-I

Answer any seven questions.

1. Solve $y'' - 2y' + y = 2\,\text{Cos}\,x$. Find the values of the arbitrary constants assuming $y(0) = 0$ and $y'(0) = 0$. 5

2. The differential equation $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1) = 0$ has a solution of the form $y = \Sigma_r\, a_r\, x^{K-r}$. Find the values of K. Find the relation between the constants for both values of K. 5

3.a) Prove that $\int_{-1}^{+1} P_n(x)\, P_m(x)\, dx = 0$ if m$\neq n$ where $P_n(x)$ and $P_m(x)$ are Legendre polynomials of order n and m respectively. 3

b) Show that $P'_{n+1}(x) - P'_{n-1}(x) = (2n+1)P_n(x)$. 2

4. Find out the condition that the vectors $\vec{A}$, $\vec{B}$ *and* $\vec{C}$ are linearly independent. Interpret the result physically. 4+1

5. For a scalar $\phi = 3x^2 + 4xy + 5xz^2$ find out the rate of change of ϕ along the direction specified by the vector $3\vec{i} + 2\vec{j} + 5\vec{k}$ at the point (1,2,3). What is the direction along which the rate of change is maximum ? 5

6. Establish the result $div\ \vec{A} = \lim \frac{\int \vec{A}\cdot d\vec{s}}{\Delta v}$, the symbols having their usual meanings. 5

7.a) Let z be defined implicitly through the equation, $f(x,y,z) = 0$

Prove that, $\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$

b) If , $z = f(ax+by)$, show that $b.\frac{\partial z}{\partial x} - a\frac{\partial z}{\partial y} = 0$. 3+2

8. State Fourier's theorem clearly stating the Dirichlet conditions. 5

9. From the Fourier expansion $f(x) = \frac{a_0}{2} + \Sigma_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$

Show that average of $[f(x)^2]$ over $(-\pi,\ \pi) = \frac{a_0^2}{4} + \frac{1}{2}\Sigma_{n=1}^{\infty}(a_n^2 + b_n^2)$ 5

10. Prove that the following relation involving the Delta function is valid $\delta(nx) = \frac{1}{|n|}\delta(x)$. Using the above relation, show that Delta function is an even function. 4+1

SECTION-II

Answer any three questions.

11. a) What is optical path ? State Fermat's principle. 1+1

b) When a light source is placed at the focus of a parabolic mirror, show that all rays after reflection go parallel to axis of mirror. What is aplanatic surface ? 2+1

12.a) Define principal and nodal points of an optical system. Show that principal and nodal points of an optical system will be identical if the media on both sides are same. 2+ 1

b) Deduce Helmholtz equation. 2

13. A small object is placed 50 cm from the curved surface of a glass hemisphere of radius 12 cm and n = 1.5. Find the cardinal points and nature of the image. 3+2

14. Find the achromatic condition for the two lenses of different focal lengths placed at a distance a apart. 5

15. What are spherical aberration and chromatic aberration ? Mention some remedy for spherical aberration. 3+2